Strut Design

THE PROBLEM

Design the lightest possible simply supported strut that is L long and capable of supporting an axial load of P without shortening more than δ. Its strong material is linear elastic with a Young's modulus of E. No part of the strut must have a thickness less than t. See Fig 1.

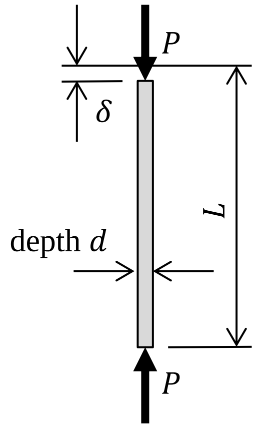


Figure 1: Strut design problem

THE SOLUTION

This is a shape design problem, one that can be transformed into a material selection problem by invoking the concept of material dilution[1]. Consider how this strut design problem would be altered if the material to be used could be supplied in a range of densities, from the most dense, with a density of ρ and Young's modulus of E, to less dense varieties, which could have properties ρ/i and E/i, where i is a dilution factor, dimensionless, which may take any value greater than 1. The less dense materials could be achieved by forming cylindrical voids in the parent material parallel to each other in the principal direction, along its 'grain', as shown in Fig 2. A member made of one of these structured materials would have two sectional areas, a gross area $A' = d^2$, and a nett area $A = A'/i$, the area of the actual material, shaded in the figure. The diluted material, considered to solidly fill the shape of the convex square member, may be referred to as a 'macromaterial[2]' by virtue of the macroscopic scale of its internal structure. This macromaterial is less stiff and less dense than its parent material by this factor of i, a factor that depends on the number and size of the voids, but which is not limited, except that it must be greater than 1. The existence of a choice of macromaterials transforms the design process from finding a complicated shape for a prescribed material to choosing a material to fill a simple convex envelope shape, choosing from a full spectrum of materials having stiffness $E' = E/i$ and density $\rho' = \rho/i$.

As Fig 2 illustrates, the effect of diluting material in this way is simply to disperse material uniformly into space to achieve a lighter, less stiff material. For transmitting load in the principal direction there is no loss of efficiency. The average axial strain in this strut is $\varepsilon = \delta/L$, which is the same for the parent material as it is for the macromaterial. The required area of parent material, A, is

$$A = P/E\varepsilon$$

All that is required in design is to disperse this amount of material into an envelope wide enough for the strut not to buckle. This simple idea can be applied to choosing the practical engineering shapes like angles, I-sections, tubes and lattices.

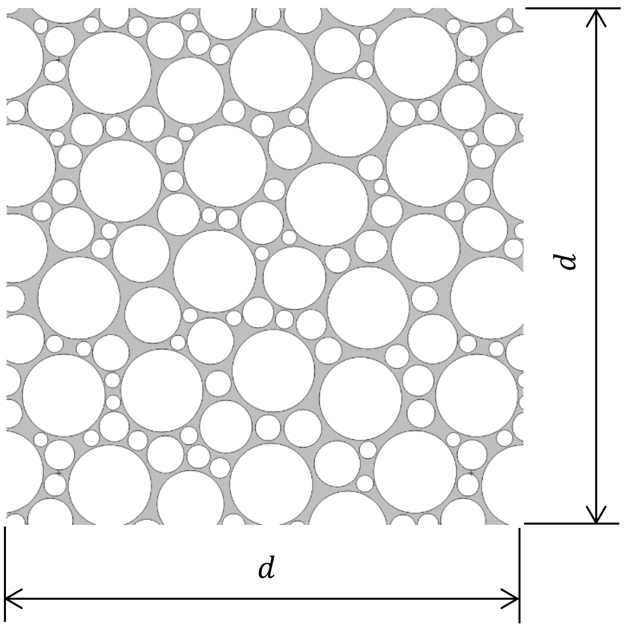


Figure 2: Cross-section through a square member of voided material

The two unknowns are d, the depth of the square section, and E'. Introducing E' as a variable brings internal (concave) shape into consideration, now becoming a part of the solution. The sectional area and inertia of the solid representative section are

$$A' = d^2$$

$$I' = \frac{d^4}{12} = \frac{A'd^2}{12}$$

Suppose the strut is on the point of buckling with a strain of ε:

$$P = E'A'\varepsilon \qquad \text{Equilibrium}$$

$$P = \frac{\pi^2 E' I'}{L^2} \qquad \text{Buckling, using Euler's equation}$$

Eliminating P and $E'A'$:

$$\varepsilon = \frac{\pi^2 d^2}{12L^2}$$

i.e. $$d = \frac{2\sqrt{3}}{\pi} L\sqrt{\varepsilon} \qquad (1)$$

This shows that the strut's slenderness, L/d, is determined solely by the buckling strain, ε; their relationship is independent of the material, which is significant for designers: a high compressive strain requires stocky members, whatever their construction, whether it is a small strut made of solid steel, a larger one consisting of a steel tube, or yet a larger one, a lattice of steel tubes; their representative envelopes will be stocky, determined precisely by d from Eqn 1.

Eliminating d from the equilibrium equation

$$P = E'A'\varepsilon = E'd^2\varepsilon = \frac{12}{\pi^2} E'L^2\varepsilon^2$$

So $$E' = \frac{\pi^2}{12} \frac{1}{\varepsilon^2} \frac{P}{L^2} \qquad (2)$$

This is the stiffness of the macromaterial required to fill the d by d section if the strut is not to buckle under the design load at the required strain. A macromaterial like that shown in Fig 2 would need a dilution factor of $i = E/E'$, i.e. a net sectional area of $A = d^2/i$.

Increasing both d and the dilution factor, i, while maintaining the value of A, would also be a minimum mass solution, but dilution comes at a price: the space is free but the complex surfaces surrounding the space are not. Increasing the dilution reduces the thickness of its parent material, and the form of Fig 2, while excellent for illustrating the principle of dilution, has practical disadvantages, with its intricacy and its thin ligaments of material. The disqualifying feature of this form is that it is multicellular. Engineers never use multiple cells when one would do. In this article, macromaterial properties are calculated for three practical engineering forms, square and cylindrical tubes, and lattices, which can be used with the two strut design equations 1 and 2 above.

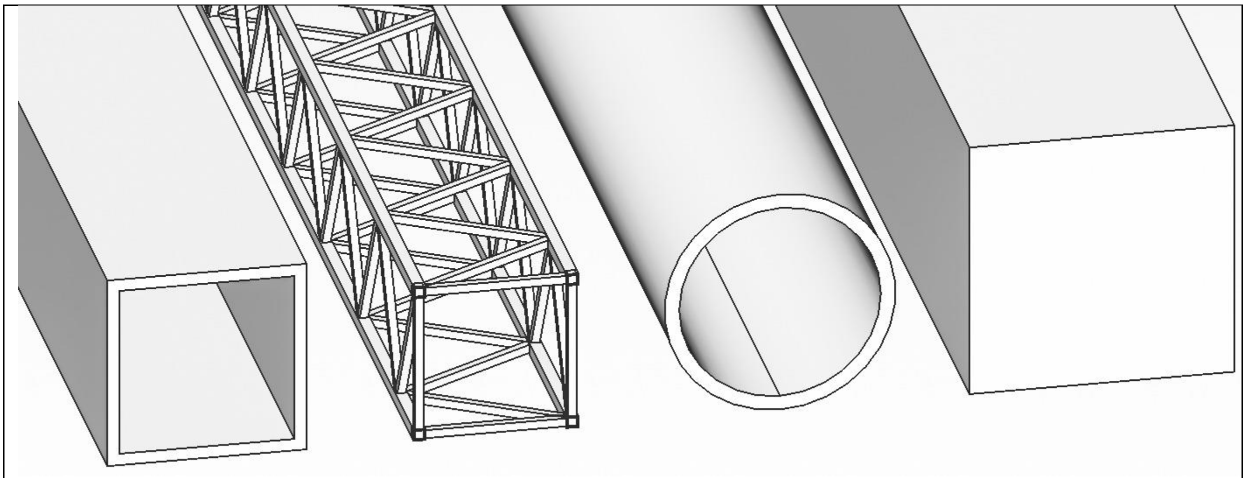
If the stiffness requirement were removed from the strut design problem, then the average strain, ε, would become a design variable. The design would be repeated for many values of ε from which the lightest solution would be selected.

The next section explains in more detail how macromaterial properties are calculated.

MATERIAL TRANSFORMATIONS

Raw materials are likely to be too stiff and heavy for a solid square sectioned strut to be appropriate in many situations, situations that are characterised by low values of P/L^2, a coefficient described by H L Cox as a 'structure loading coefficient'[1], which appears in Eqn 2. The lower the value of P/EL^2, the more space will be required around this material to produce a macromaterial with a suitably low value of E', which occurs when engineers synthesize macromaterials using repeating shapes, or forms, like the three shown in Table 1. A principal parameter for each of the three forms is slenderness, broadly speaking, the ratio, λ, of the sections overall size, d, to its material thickness, t. Increasing the slenderness increases the proportion of space enveloped and hence the dilution factor, i.

Table 1: Four members



Square hollow section	Lattice with 60° bracing	Circular hollow section	Solid section
$\lambda = d/t$	$\lambda = d/t$ (d = chord length)	$\lambda = d/t$	Side length $= t'$
$i = \frac{E}{E'} = \frac{A'}{A} = \frac{1}{2}\lambda$	$i = \frac{E}{E'} = \frac{A'}{A} = \frac{9}{16}\lambda^2$	$i = \frac{E}{E'} = \frac{A'}{A} = \frac{3}{2\pi}\lambda$	$i = 1$
$\eta = 1$	$\eta = \frac{1}{2}$	$\eta = 1$	$\eta = 1$
$\tau = \frac{t'}{t} = \sqrt{2}\lambda$	$\tau = \frac{t'}{t} = \frac{3}{2}\lambda$	$\tau = \frac{t'}{t} = \sqrt{\frac{3}{2}}\lambda$	

A form containing the parent material is to be equivalent to a solid member containing its macromaterial. The shape of the solid member may be square or rectangular, a simple convex shape. Its size must be such that it has the same two radii of gyration as the member it is representing. All the members depicted in Table 1 have the same radii of gyration. Suppose the sectional area of one of the forms is A and its two inertias are both Ar^2 and its material has a Young's modulus of E. If the side length (thickness) of the solid square representative shape is t', then its sectional area, $A' = t'^2$. The macromaterial will have a Young's modulus of E' which will faithfully represent the axial stiffness, so that

$$EA = E'A'$$

Since both sections have the same radii of gyration

$$EAr^2 = E'A'r'^2$$

confirming that their flexural stiffness characteristics are also identical.

Hence $r^2 = \frac{t'^2}{12}$

and $E' = \frac{A}{A'}E = \frac{A}{12r^2}E$

The dilution factor is defined as the ratio of the two material stiffnesses

i.e. $i = \frac{E}{E'} = \frac{A'}{A} = \frac{12r^2}{A}$

This illustrates the nature of material transformations. i is the first of several dimensionless factors to be introduced that define a macromaterial's properties in terms of the material properties of its parent material, as in

$$E' = E/i$$

Other factors that are shown in Table 1 are the efficiency, η, and the thickness ratio, τ. These are defined as

$$\eta = \frac{E'}{\rho'} \Big/ \frac{E}{\rho}$$

where ρ and ρ' are the two material densities and

$$\tau = \frac{t'}{t}$$

where t and t' are the two material thicknesses. These three ratios are presented as functions of each form's slenderness in Table 1, where it is assumed that the slenderness, λ, is large.

The dilution factor can never be less than one. By shaping a material there is no way it can be made stiffer. Likewise, the efficiency factor can never be more than one. Perfect efficiency, with $\eta = 1$, is a characteristic of all prismatic or extruded forms in their principal direction because all the parent material is responding fully to the average strain in this direction. But a typical lattice has an efficiency, η, of about 0.5 because about half of the parent material, all that is contained in the bracing, remains practically unstressed by uniform strain in the principal direction.

So, a basic set of material transformations are:

$$E' = E/i$$

$$\rho' = \rho/\eta i$$

$$t' = \tau t$$

The compressive strength, σ', is going to be a function of both σ, the compressive strength of the parent material, and G, the shear modulus of the parent material, as well as the parent material's Young's modulus, E. The shear modulus, G', of the macromaterial will be a function of both E and G. The torsional stiffness of the representative member may also need to be calculated.

Table 2: Form factors

	Square hollow section	Lattice	Circular hollow section
$i =$	$\frac{0.951}{\sqrt{\varepsilon_B}}$	$\frac{0.703}{\varepsilon_B}$	$\frac{0.577}{\varepsilon_B}$
$\eta =$	1	0.5	1
$\tau =$	$\frac{2.69}{\sqrt{\varepsilon_B}}$	$\frac{1.68}{\sqrt{\varepsilon_B}}$	$\frac{1.48}{\varepsilon_B}$

Instead of working with compressive strengths, it is advantageous to work with a compressive failure strain, ε, since this is a primary design parameter, as has already been shown. In Appendix 2, material transformations are calculated for the three forms of Table 1, and their form factors are summarised in Table 2 above.

The form factors are expressed in terms of the form's buckling strain. Notice that each of the three forms have their distinct characteristics, some form factors varying inversely with ε_B and others inversely with $\sqrt{\varepsilon_B}$. They plot as straight lines on charts with log scales, as shown in Fig. 3. The numbers on the chart indicate values of τ, the thickness ratio. These increase with increasing slenderness and result in increasing dilution and decreasing strength. Lattices major on dilution for a given slenderness, recognised by them being referred to as space structures. Circular hollow sections major on buckling strength. These characteristics, which are well known qualitatively, can be quantified by material dilution theory.

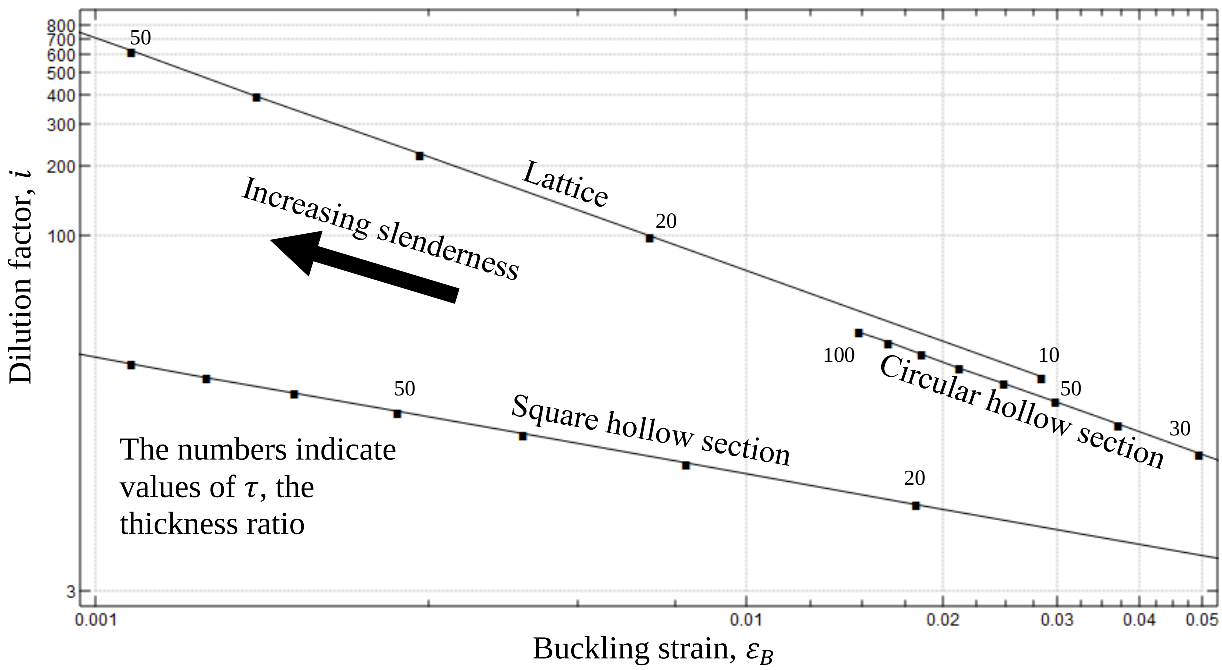


Figure 3: Dilution factor, i, versus buckling strain, ε_B, for the three forms

FORM SELECTION

When the parent material is specified, selecting a macromaterial is the same as selecting a form. For this investigation there are just three generic forms to choose from and for each, a choice of slenderness. The choice is based on what value of the dilution factor is appropriate for a given design strain and this can be explored on a chart like that of Fig 3. The optimal value of i is defined by Eqn 2 which can be rewritten

$$\frac{1}{i} = \frac{\pi^2}{12}\frac{1}{\varepsilon^2}\frac{P}{EL^2}$$

where the right-hand term, P/EL^2, is the data that is defining the design requirement. This equation defines a 'target line' on an i versus ε chart, indicating where the design stiffness and strength requirements are both precisely met. Fig 4 shows a set of these target lines overlaid on the chart of Fig 3. The horizontal axis has been relabelled 'design strain'. Since practical considerations are likely to mean that the final design point will not lie exactly at the most desired point, the target line functions as a regional boundary. The permissible region for a design point is below and to the right of one of these target lines and to the left of the vertical permissible strain target line. The chosen design point must lie on a line representing a form.

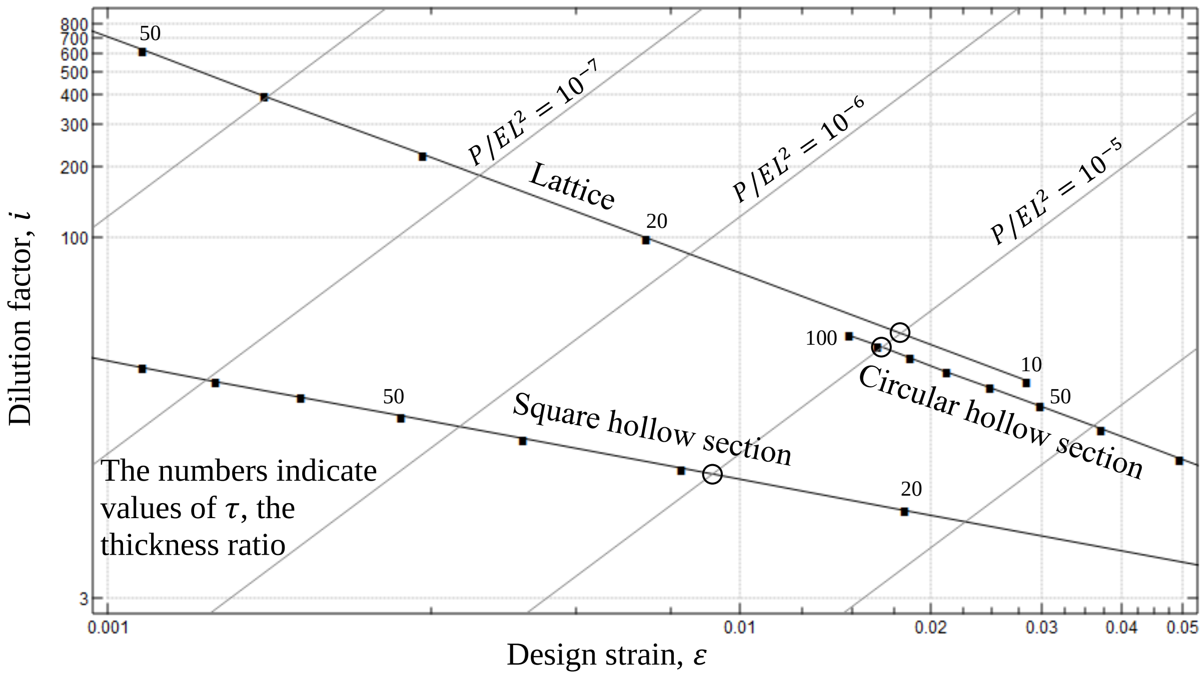


Figure 4: Fig 3 with contours of P/EL^2 superimposed

Fig 4 shows three potential designs marked by the ringed points for $P/EL^2 = 10^{-5}$. The design strains are 0.009, 0.017 and 0.018. All these struts are stocky having t' between $0.1L$ and $0.15L$. The lattice has $\tau = 13$ while the circular hollow section has $\tau = 90$ illustrating how plate thickness is much smaller than lattice member thickness for a comparable design.

But which is the lightest? Fig 4 can be replotted with the density ratio, ηi, in place of i, plotted up the vertical axis. For the two hollow sections η is 1, so their lines have not moved. But the lattice, with $\eta = 0.5$, has dropped by this factor, and is shown by the dashed line. To compare densities, the lighter density is the higher design point on the chart. The design point for the circular hollow section shows that it gives a solution with a lighter density than that of the lattice

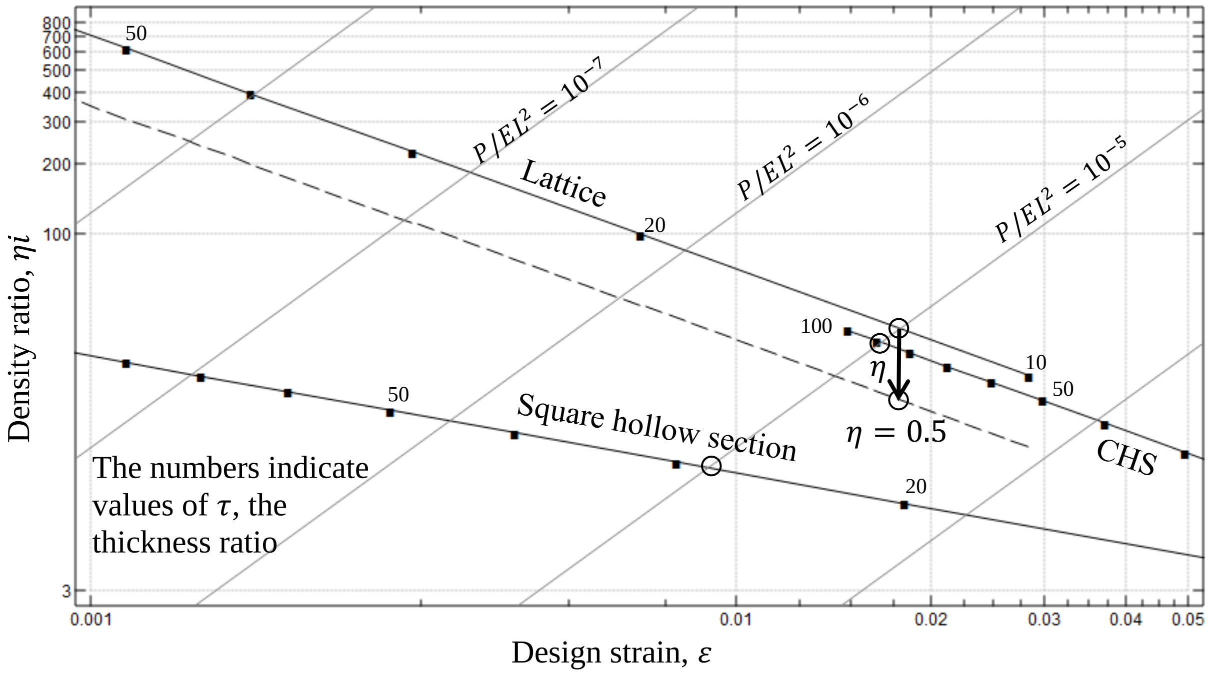


Figure 5: Fig 4 with density ratio factor, ηi, not i, up the vertical axis

because it is above it, and, because it is operating at a lower strain, it has a smaller gross thickness, t', so is lighter on account of that also.

Graphical representations are useful for appreciation, but results are better obtained by computation, as in the following example:

$L = 200$ mm
$P = 450$ N
$t = 1.5$ mm
$E = 1650$ N/mm^2 SLS nylon
$\rho = 0.95 \times 10^{-3}$ gm/mm^3

There is no stiffness requirement. The target is a strut of minimum mass using one of the three forms.

As a starting point it is useful to check out the undiluted solution, the lightest strut made of solid parent material. Its buckling strain is ε_1 where, from Eqn 2

$$\varepsilon_1 = \pi\sqrt{\frac{1}{12}\frac{P}{EL^2}} = 0.00237$$

From Eqn 1, the depth of section

$$d_1 = \frac{2\sqrt{3}}{\pi}L\sqrt{\varepsilon_1} = 10.7 \text{ mm}$$

The mass of this strut

$$m_1 = \rho {d_1}^2 L = 21.9 \text{ gm}$$

This is the optimum design for a design strain of 0.00237. It is the design for which no dilution is necessary. Using a tube with the same cross-sectional area would satisfy the design requirements, but the additional complexity would not be warranted. It is always possible to increase dilution by adding voids or stiffeners, but unless they are necessary, they are unwarranted complexity. Dilution is minimised if the design strain defines the global buckling strain. This gives the smallest possible representative solid section and the least dilution and maximises the parent material thickness. It is not generally known what value of the design strain will give the optimum design, so ε is taken as a design parameter. For each of a range of values of ε, calculation proceeds in 6 steps:

1. $A = P/E\varepsilon$ — Sectional area of parent material
2. $A' = t'^2 = \frac{12}{\pi^2}\varepsilon L^2$ from Eqn 1 — For global buckling to occur at ε
3. $i = \frac{A'}{A}$ — Dilution factor
4. Calculate ε_B, η and τ using this value of i and the data in Table 2

Provided $\varepsilon_B > \varepsilon$, local buckling does not occur, so continue:

5. $t = t'/\tau$ — Thickness of parent material
6. $M = \rho AL/\eta$ — Mass of member

This provides a candidate solution for every value of the design strain. The solution with $t > 1.5$ that gives the least mass is the winner.

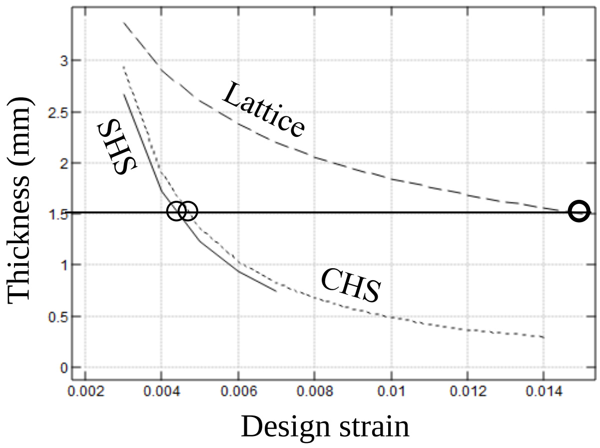


Figure 6: Thickness v. design strain

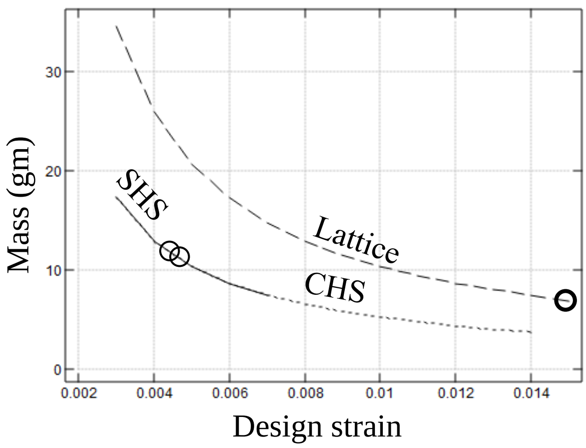


Figure 7: Mass v. design strain

Figures 6 and 7 show the results from this set of calculations. The right-hand ends of all the curves are where local buckling strains catch up and would overtake the global buckling design strain. The square hollow section drops out before the circular hollow section because flat plates are more vulnerable to local buckling than shells. The SHS and the CHS have identical masses for design strains that do not cause local buckling, and half the mass of the lattice solution at the same design strain because of the lattice's bracing, which weighs as much as its chords without bearing any of the axial load. For any design strain the thickness of the lattice member is several times greater than the wall thickness of the tubes, which is why, in this particular example, the lattice wins with a mass of 7 gm and a thickness of 1.5 mm, illustrated by the bold rings in the figures. The best that tubes can offer is a mass of 11 gm, all lighter than the 22 gm of the solid rod.

The value of τ for the lattice relates to the chords. The thickness of the bracing is $\sqrt{2}$ times smaller, and this has not been overlooked in plotting the lattice's line in Fig 6, where thickness refers to the thickness of the bracing.

This design example is characterised by $P/EL^2 = 7 \times 10^{-6}$, when the choice between the shell and the lattice form is stark. There is no solution in between. As P/EL^2 decreases, more dilution is required and, provided the scale is large enough, a hierarchy of forms is economical. Lattices of tubes are common for footbridges and for offshore oilrigs. The Forth Bridge is a lattice of lattices, as is the Eiffel Tower. Material dilution theory is the appropriate vehicle for handling hierarchies of form. Consider a strut composed of a lattice of tubes. There are four failure modes, each with their failure strains:

1. Material failure, i.e. yield strain ε_m
2. Tube wall buckling ε_t
3. Chord buckling of the lattice ε_l
4. Global strut buckling ε_s

The failure modes are effectively in series, in that the failure at one hierarchical level will cause failure of the levels above it, where 'above' in this context considers the parent material to be at the lowest level and the whole strut to be at the highest level. It is reasonable for a designer to prescribe $\varepsilon_m > \varepsilon_t > \varepsilon_l > \varepsilon_s$ without making them all equal. The reason is that the lower level elements are more susceptible to abuse and damage than the higher level elements because accidental loading is more likely to fail the overall structure by first damaging the lower levels, e.g. by bending a lattice member or denting a tube. With multiple hierarchies of form the design process remains similar to that performed with the 6 steps, except that the choice will be of a macromaterial that has been transformed twice, from steel, say, to tubular steel, and from tubular steel to latticed tubular steel. The form factors multiply up together, as shown in Table 3; for example, the overall dilution factor is $i_t i_l$, the product of the dilution factors for each of the two forms.

Table 3: Three hierarchical levels of material, two levels of form

	Young's modulus	Strength	Density	Thickness
Steel	E	$E\varepsilon_m$	ρ	t
Tubular steel	$E' = E/i_t(\varepsilon_t)$	$E'\varepsilon_t$	$\rho' = \rho/\eta_t i_t(\varepsilon_t)$	$t' = t/\tau_t(\varepsilon_t)$
Latticed tubular steel	$E'' = E'/i_l(\varepsilon_l)$	$E''\varepsilon_l$	$\rho'' = \rho'/\eta_l i_l(\varepsilon_l)$	$t'' = t'/\tau_l(\varepsilon_l)$

This is structural design theory. Every design must still be rigorously analysed and tested and subjected to the scrutiny of design standards.

FURTHER CONSIDERATIONS

Other forms

Just three basic forms have been considered. Other forms include the many sections, I-sections, angles, T-sections, and the various bracing geometries of lattice members. Of interest is the Vierendeel truss, a truss with transversals but no diagonals, disadvantaged by its great flexibility in shear. All possible geometries can be offered as players on a level playing field for assessment by material dilution theory.

Then there is another whole family of forms, dispersing material in 2D, for decking and roofing, which include corrugated sheet, grids, cellular slabs and lattice decks. Each of these forms has its own material transformation factors.

The principles of material dilution can be applied more generally as material substitution for composite materials. Any composite material can be treated as a macromaterial.

Prestress can transform two materials' properties; mixing strong tensile materials with strong compressive materials and prestressing one against the other can result in high performance.

Tapered struts

A genuinely pin jointed strut should be tapered to avoid the unwanted inclusion of two heavy end plates. A preferred shape for tapered members is described in [4]. Its profile is defined by the equation

$$y\frac{d^2y}{dx^2} = \text{constant}$$

Its feature is that its elastic buckling mode shape also follows this profile. Its buckling load is

$$P = \frac{2\pi EI}{L^2}$$

where I is the inertia at its middle section. The member has constant cross-sectional area along its length. Form selection is focused on the middle section.

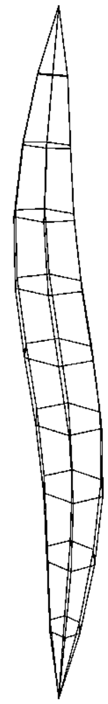


Figure 8: Shear buckling mode of a tapered strut

Shear buckling

Shear buckling has not been considered in this article. Shear flexibility reduces buckling strains. When a member's shear stiffness is low, as when the bracing of a lattice has been minimised or when diagonal bracing is missing, and when buckling strains are large, shear flexibility should be accounted for as a reduction in a macromaterial's compressive strength. This will be dealt with in detail in a separate article on material transformations. Fig 8 shows how shear flexibility can produce an S-shaped buckling mode in a tapered strut.

Cost

Minimum mass is a convenient target for theoretical exercises, but minimum cost is a more practical target, and cost per unit mass is a macromaterial property which, if appropriate data were available, could be included in the design optimisation procedure. The cost factor would increase for every hierarchical step and would quickly reveal that many lightweight solutions would be financially unattractive. Which design is more financially viable changes with every technological development; what has been dismissed as non-viable may become viable with the invention of new manufacturing techniques. Changing cost factors will change the shape of our designs.

Rectangular or elliptical sections?

The section with the least shape is the ellipse, against which the rectangle has a dilution factor of $\pi/3 = 1.04$. What shape is chosen for representative members is of no consequence provided whatever shape is chosen is always remembered and always used for comparison in subsequent calculations.

CONCLUSION

Designers are preoccupied with choosing shape, guided by experience and analysis. Material dilution theory transforms the design process into choosing materials to solidly fill a simple envelope shape. The theory is relevant where buckling is a governing criterion.

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APPENDIX 1: LOCAL BUCKLING STRAINS

Local buckling of the lattice

The Euler buckling formula applies to the local buckling of the lattice member of Table 1 as it buckles in the mode illustrated in Fig A1.1. The chords buckle into segments of a sinusoidal wave. Because the chords are rotationally restrained by the bracing, the characteristic half-wavelength, l, is shorter than the nodal spacing, d. With the geometry of the lattice member in Table 1

$$l = 0.81d$$

so, the local buckling strain of the lattice

$$\varepsilon_B = \frac{\pi^2}{12 \times 0.81^2} \frac{t^2}{d^2} = \frac{1.25}{\lambda^2}$$

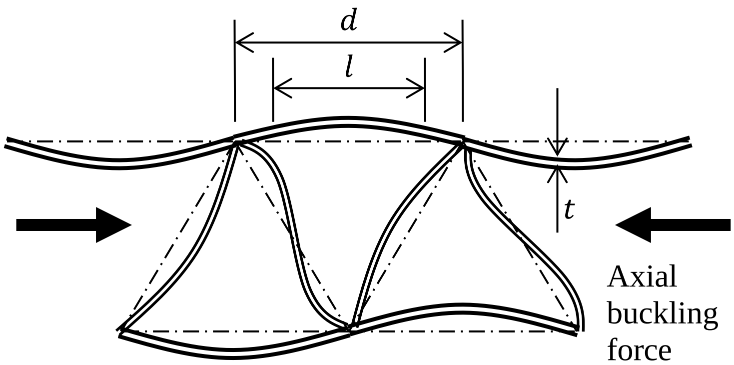

Figure A1.1: Local buckling of the lattice

Local buckling of the square tube

The buckling mode for a simply supported rectangular flat plate, d wide, has a characteristic wavelength equal to the spacing, d, of its (long) simply supported edges, so whether a plate has four faces, like the square hollow section of Table 1, or is part of a tube having an n-sided polygonal cross-section, like that shown in Fig A1.2, provided n is an even number, adjacent $d \times d$ panels buckle in a chessboard fashion, none of them providing flexural support to their adjacent panels because one that is bowing out is adjacent to ones that are bowing in. For a panel width of d and a plate thickness of t, the buckling strain is[5]

$$\varepsilon_B = \frac{\pi^2}{3(1-\nu^2)} \frac{t^2}{d^2}$$

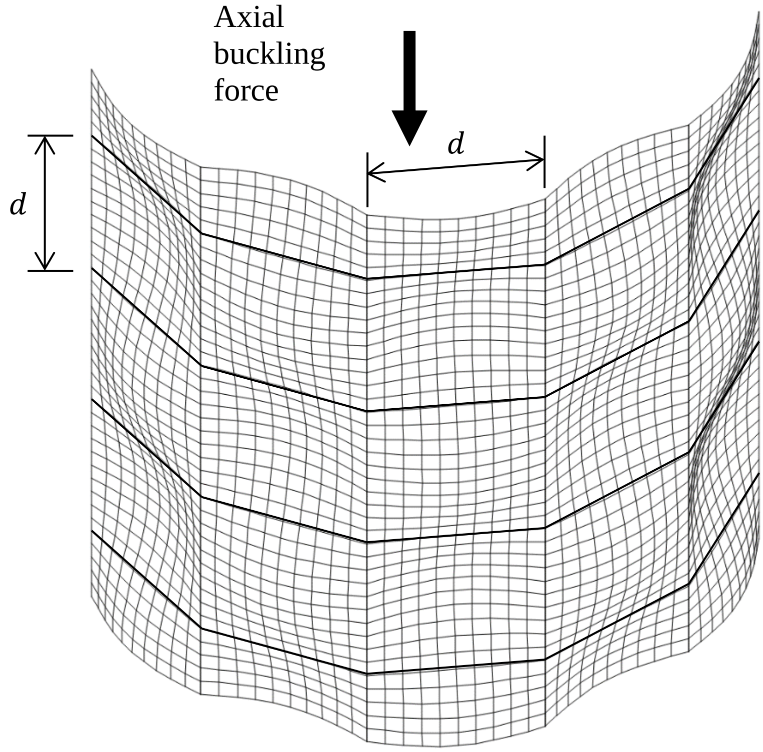

Figure A1.2: Local buckling of long flat plates

Taking the Poisson's ratio, ν, of the panel's material as 0.3, the local buckling strain of the square hollow section

$$\varepsilon_B = 3.62 \frac{t^2}{d^2} = \frac{3.62}{\lambda^2}$$

Local buckling of the cylindrical tube

Cylindrical shells compressed axially can buckle in a longitudinal mode, with a sinusoidal wave in the axial direction having a half-wavelength of approximately $1.22\sqrt{dt}$, giving a buckling strain of [6]

$$\varepsilon_B = \frac{2}{\sqrt{3(1-\nu^2)}}\frac{t}{d}$$

Taking the Poisson's ratio, ν, of the panel's material as 0.3, the local buckling strain of the circular hollow section

$$\varepsilon_B = 1.21\frac{t}{d} = \frac{1.21}{\lambda}$$

This is inversely proportional to λ, not λ^2, indicating how shells are more resistant to buckling than flat plates, a significant fact for designers. However, thin shells, with high values or λ, are sensitive to flat spots, and can buckle at lower strains in a faceted mode like that of Fig A1.2. Then ε_C can be based on modelling n potential facets around its circumference, when

$$\varepsilon_B = \frac{n^2}{3(1-\nu^2)}\frac{t^2}{d^2}$$

This formula may be more appropriate for large values of λ, n being determined by assessing shape imperfections.

APPENDIX 2: BASIC MATERIAL TRANSFORMATIONS

Square hollow section

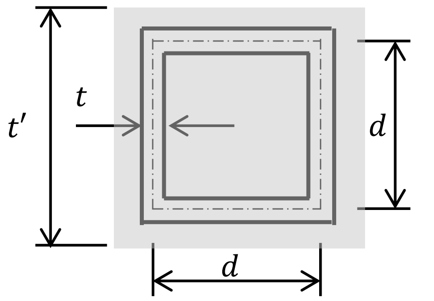

Figure A2.1: Square hollow section

Mid-surface side length $= d$
Wall thickness $= t$

$$A = 4dt$$

$$I = \frac{d^3t}{2} + \frac{d^3t}{6} = \frac{2}{3}d^3t$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{2}{3}\frac{d^3t}{4dt}} = \frac{d}{\sqrt{6}}$$

Hence $\frac{t'}{\sqrt{12}} = \frac{d}{\sqrt{6}}$ and $t' = \sqrt{2}d$

Hence $i = \frac{t'^2}{A} = 2\frac{d^2}{4dt} = \frac{1}{2}\frac{d}{t} = 0.500\lambda$

This being a flat plated form

$$\varepsilon_B = \frac{3.62}{\lambda^2}$$

Hence $i = 0.500\frac{\sqrt{3.62}}{\sqrt{\varepsilon_B}} = \frac{0.951}{\sqrt{\varepsilon_B}}$

Being an extruded form:

$$\eta = 1$$

$$\tau = \frac{t'}{t} = \sqrt{2}\frac{d}{t} = 1.41\lambda = 1.41\frac{\sqrt{3.62}}{\sqrt{\varepsilon_B}} = \frac{2.69}{\sqrt{\varepsilon_B}}$$

Lattice

The 4-chorded lattice truss is singly braced on each face with members inclined at 60° to the chords. The chord length is d and each chord has a solid square section of side length, t. Each bracing member is square with side length $t/\sqrt{2}$, so that the member's material is divided equally between the chords and the bracing. The distance between chord centrelines is $\left(\sqrt{3}/2\right)d$.

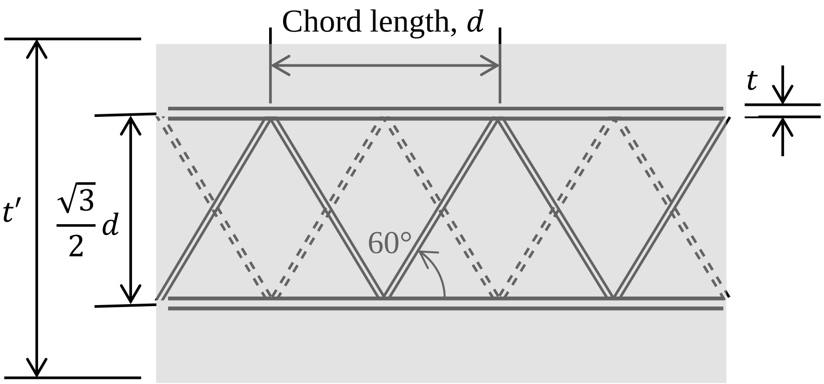

Figure A2.2: Lattice

$$A = 4t^2$$

$$r = \frac{\sqrt{3}}{4}d$$

Hence $\frac{t'}{\sqrt{12}} = \frac{\sqrt{3}}{4}d$

And $t' = \frac{3}{2}d$

Hence $i = \frac{t'^2}{A} = \frac{9}{4}\frac{d^2}{4t^2} = \frac{9}{16}\left(\frac{d}{t}\right)^2 = 0.563\lambda^2$

This being a lattice form

$$\varepsilon_B = \frac{1.25}{\lambda^2}$$

Hence $i = 0.563\,\frac{1.25}{\varepsilon_B} = \frac{0.703}{\varepsilon_B}$

Since material is divided equally between the chords and the bracing

$$\eta = 0.5$$

$$\tau = \frac{t'}{t} = \frac{3}{2}\frac{d}{t} = 1.5\lambda = 1.5\frac{\sqrt{1.25}}{\sqrt{\varepsilon_B}} = \frac{1.68}{\sqrt{\varepsilon_B}}$$

Circular hollow section

Mid-surface diameter $= d$
Wall thickness $= t$

$$A = \pi d t$$

$$I = A\frac{d^2}{8}$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{2\sqrt{2}}$$

Hence $\frac{t'}{\sqrt{12}} = \frac{d}{2\sqrt{2}}$

And $t' = \sqrt{\frac{3}{2}}d$

Hence $i = \frac{t'^2}{A} = \frac{3}{2}\frac{d^2}{\pi d t} = 0.477\lambda$

This being a curved plated form

$$\varepsilon_B = \frac{1.21}{\lambda}$$

Hence $i = 0.477\,\frac{1.21}{\varepsilon_B} = \frac{0.577}{\varepsilon_B}$

Being an extruded form

$$\eta = 1$$

$$\tau = \frac{t'}{t} = \sqrt{\frac{3}{2}}\frac{d}{t} = 1.22\lambda = 1.22\frac{1.21}{\varepsilon_B} = \frac{1.48}{\varepsilon_B}$$

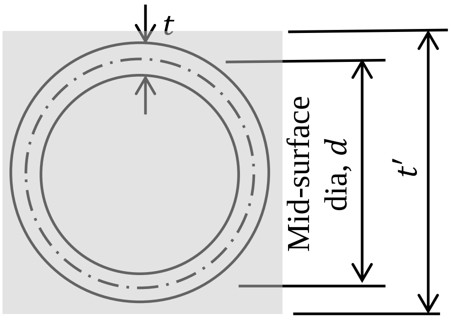

Figure A2.3: Circular hollow section